2. Motion relationships and torques

2.1 Rotation angle of a single joint
as a function of deflection angle ß

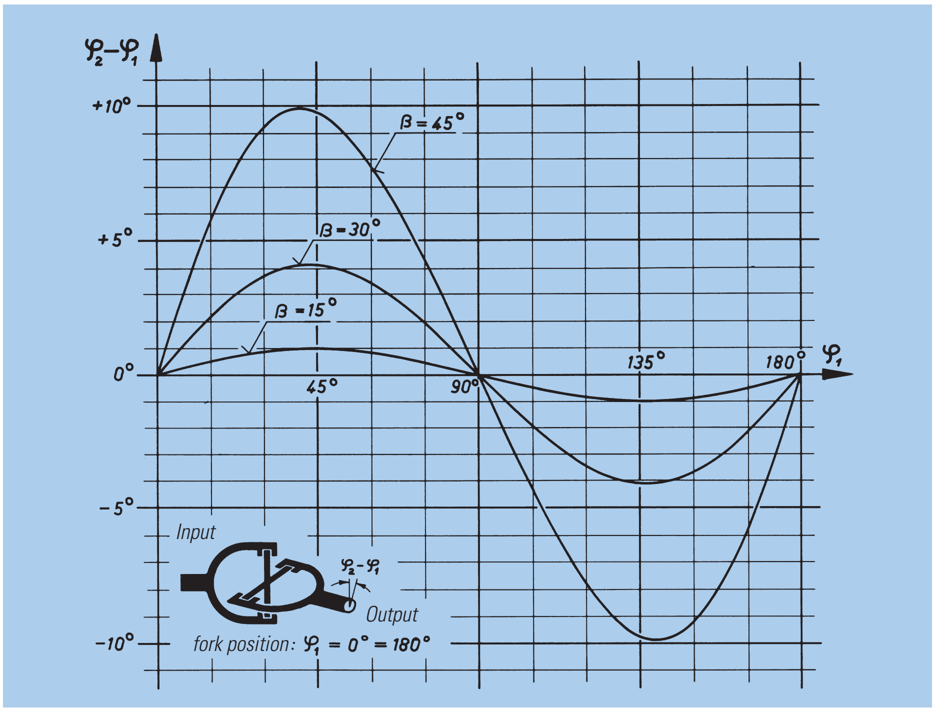
φ_1 = Input – rotation angle
φ_2 = Output – rotation angle

If a single joint is deflected by angle ß and rotated in this condition, rotation φ_2 of the output shaft differs from rotation angle φ_1 of the input shaft. The relationship between the two rotation angles is as follows:

$$\tan \varphi_2 = \frac{\tan\varphi_1}{\cos ß}$$

As can be seen from the adjacent diagram, maximum lead occurs at about 45 °, maximum lag at about 135°.

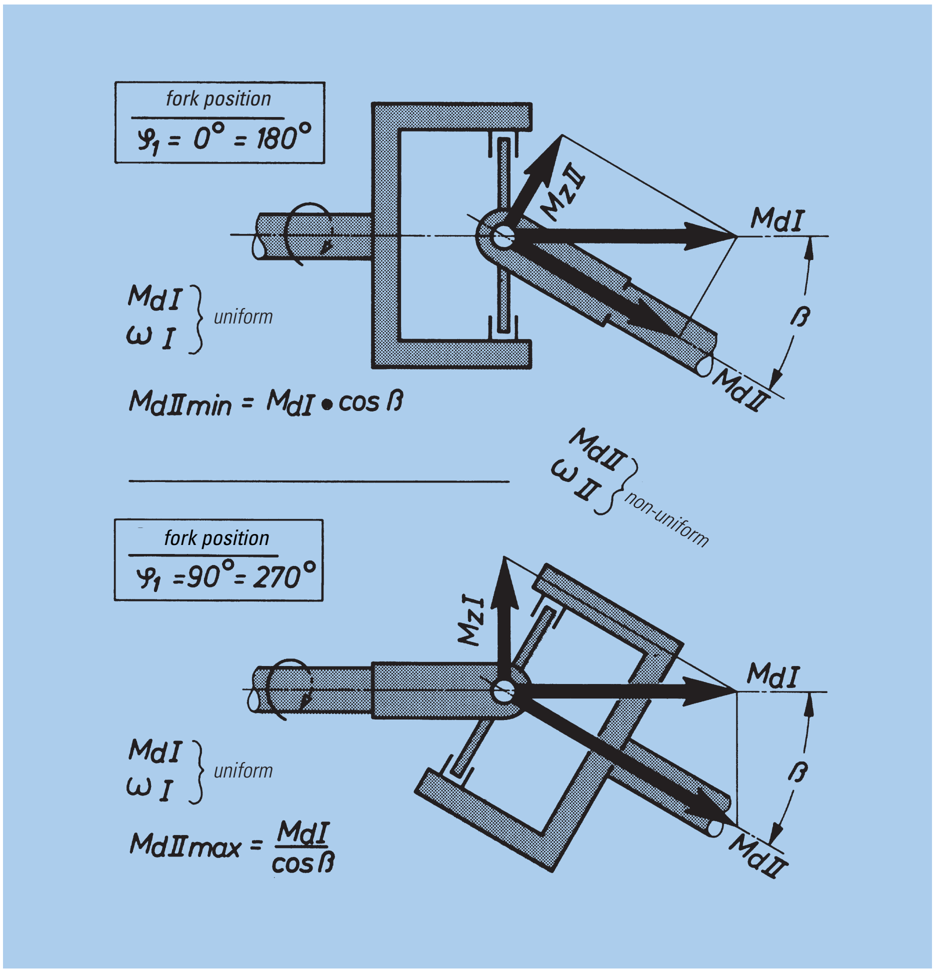
Fork position $\varphi_1 = 0°$ is then obtained, when the input fork is located in the deflection plane of the joint.



2.2 Motion and torque characteristics of a sigle joint
as a function of deflection angle ß

M_{dI} = Input torque
M_{dII} = Output torque
ω_I = Input – angular velocity
ω_{II} = Output – angular velocity

When analyzing the motion and torque characteristics of a singular joint, it is found that with a constant angular velocity- and torque input, a fluctuating motion and torque curve is obtained at the output. The reason for this fluctuation can easily be illustrated by following the torque characteristics at the fork position $\varphi_1 = 0°$ and $\varphi_1 = 90°$ as shown at left. Since the torque can only be transmitted in the spider plane, the spider however, depending on the fork position, is always at a right angle to the input or output axis, output torque fluctuates twice per revolution between $M_{dI} \cdot \cos ß$ and $M_{dI}/\cos ß$.



The transmitted power, however, is constant, if you disregard friction losses in the bearings.

Therefore, the following applies:

$$N_I = N_{II} = \text{Constant}$$

$$M_{dI} \cdot \omega_I = M_{dII} \cdot \omega_{II} = \text{Constant}$$

$$\frac{M_{dI}}{M_{dII}} = \frac{\omega_{II}}{\omega_I} = \frac{\cos ß}{1 - \cos^2 \varphi_1 \cdot \sin^2 ß}$$

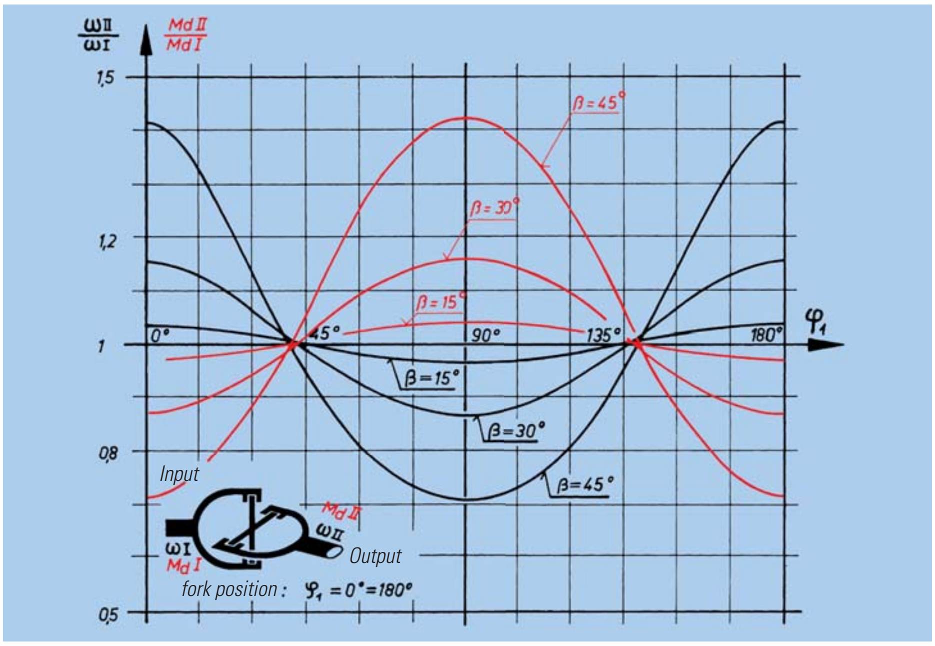
For fork position $\varphi_1 = 0°$ we obtain:

$$\frac{M_{dI}}{M_{dII\,min}} = \frac{1}{\cos ß} = \frac{\omega_{II\,max}}{\omega_I}$$

and for fork position $\varphi_1 = 90°$:

$$\frac{M_{dI}}{M_{dII\,max}} = \cos ß = \frac{\omega_{II\,min}}{\omega_I}$$

$$\frac{M_{dI}}{M_{dII}} = \frac{\omega_{II}}{\omega_I} \qquad \frac{\omega_I}{\omega_{II}} = \frac{M_{dII}}{M_{dI}}$$



2.3 Motion and torque characteristic of a universal driveline

as a function of deflection angles $ß_1$ and $ß_2$

Section 2.2 illustrates that angular velocity and torque at the output of a single joint follow a sinusoidal pattern with a 180° cycle. Maximum angular velocity $\omega_{II\,max}$ max coincides with minimum torque $M_{d\,II\,min}$ and vice versa. From this it can be deduced that a uniform output is possible, when a second joint, with a 90° phase shift is connected to the first joint by means of a shaft. Then, the non-uniform motion of the first joint can be balanced by the non-uniform motion of the second joint. The required 90° phase shift is always met, when the two inner forks happen to be in the deflection plane of their respective joints. Moreover, the two deflection angles $ß_1$ und $ß_2$ of both joints must be the same. (See also Section 1.1 and 1 .4).

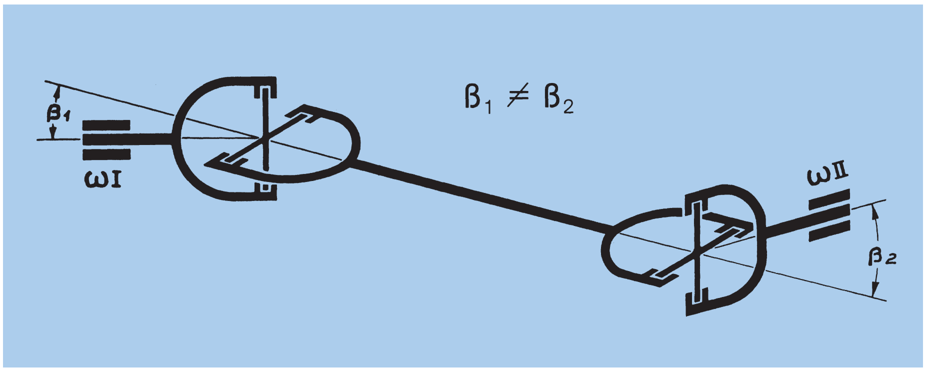
With unequal deflection angles, complete compensation is not possible.
For $ß_2 > ß_1$ the following applies:

$$\left(\frac{\omega_{II\,min}}{\omega_I}\right)_{max} = \frac{\cos ß_1}{\cos ß_2}$$

$$\left(\frac{\omega_{II\,min}}{\omega_I}\right)_{min} = \frac{\cos ß_2}{\cos ß_1}$$

$$\left(\frac{M_{dII}}{M_{dI}}\right)_{max} = \frac{\cos ß_1}{\cos ß_2}$$

$$\left(\frac{M_{dII}}{M_{dI}}\right)_{min} = \frac{\cos ß_2}{\cos ß_1}$$



3. Fluctuation rate

3.1 Single joint

As explained under 2.1, on a single joint the output velocity deviates from the input velocity. This means, the speed ratio is not uniform. This non-uniformity (fluctuation) can be calculated as a dimensionless value:

Fluctation rate

$$U = \frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_1} = \frac{1}{\cos ß} - \cos ß$$

3.2 Universal driveline (2 joints connected in series)

If the preconditions listed in Chapter 1 for obtaining a complete motion compensation cannot be met, it must be aimed for that: $U \leqq 0{,}0027$.

3.3 Universal driveline with more than two joints

Design requirements might dictate the use of a universal driveline that employs more than 2 joints. This universal driveline, however, must then incorporate an intermediate bearing.
Here, also, the condition applies:
$U_{Res} \leqq 0{,}0027$.
Here, U_{Res} expresses the total fluctuation of the driveline.
Observe, when determining U_{Res}:

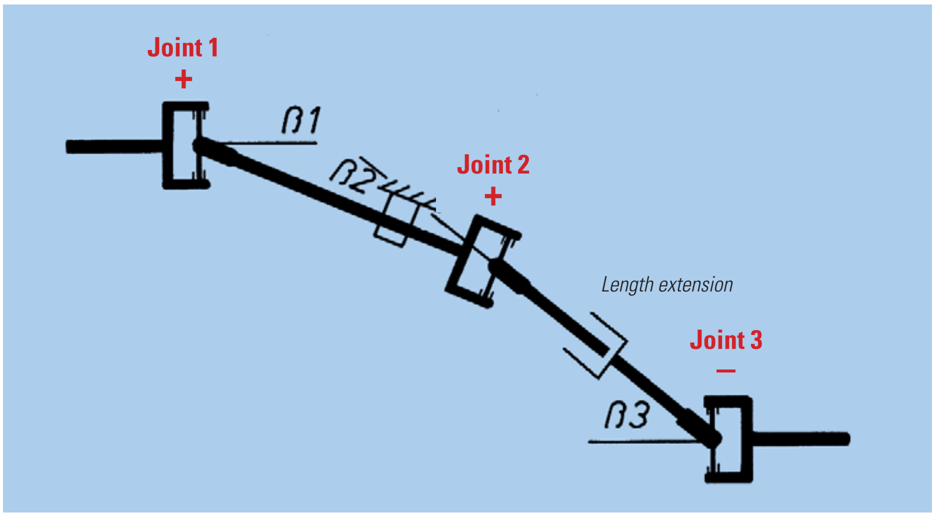
a) Joints with the same fork position get the same sign.
b) The fluctuation rate of each joint must be calculated individually U_1, U_2, U_3.
c) The signs must be observed when adding:

$$U_{Res} = \pm U_1 \pm U_2 \pm U_3$$

Since the rate of fluctuation is a function of deflection angle ß, a limiting condition can be set in regard to the resulting deflection angle $ß_{res}$

$$ß_{res} = \sqrt{\pm ß_1^2 \pm ß_2^2 \pm ß_3^2} \leqq 3°$$

$ß_{res}$ corresponds to the deflection angle of a single joint if it were to replace the entire driveline.



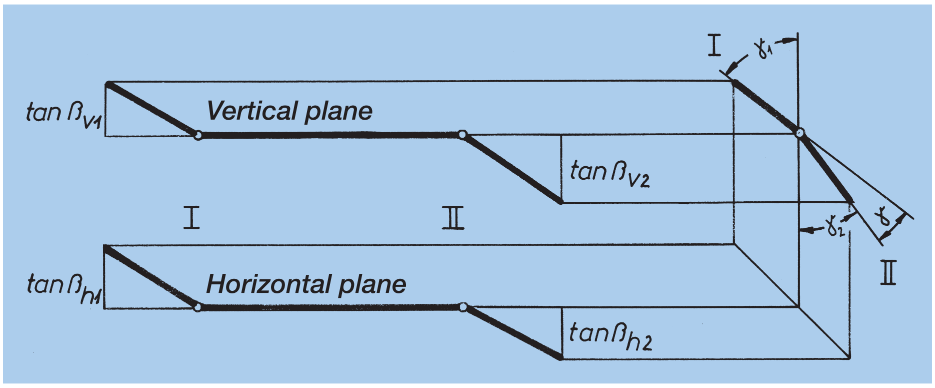
4. Offset angle

On drivelines with three-dimensional deflection angles, input and output shaft are not located in one plane. This results, if no special measures are taken, in a non-uniform output motion. The constantly repeating acceleration and deceleration unleashes inertia forces which can greatly reduce the life of the joints. However, not only the driveline, the driven equipment also is subjected to these forces and vibration caused by them. To avoid this, the inner forks must be offset relative to each other such that each fork ends up in the plane of deflection of its joint. The angle between both deflection planes is called offset angle γ and it can be obtained as follows.

Example 1

$$\tan \gamma_1 = \frac{\tan ß_{h1}}{\tan ß_{v1}} \quad ; \quad \tan \gamma_2 = \frac{\tan ß_{h2}}{\tan ß_{v2}}$$

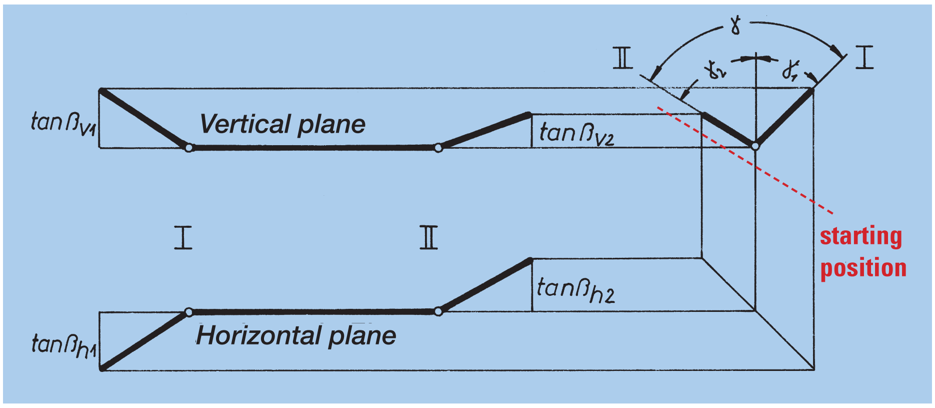
Offset angle $\gamma = \gamma_1 - \gamma_2$



Example 2

$$\tan \gamma_1 = \frac{\tan ß_{h1}}{\tan ß_{v1}} \quad ; \quad \tan \gamma_2 = \frac{\tan ß_{h2}}{\tan ß_{v2}}$$

Offset angle $\gamma = \gamma_1 + \gamma_2$



As shown by the graphic illustrations, in both examples two directions of rotation are possible:

Example 1:
a) Rotate joint 1 counter clockwise by the offset angle
b) Rotate joint 2 clockwise by the offset angle.
The direction for viewing is, in both cases, from joint 1 to joint 2.

Example 2:
a) Rotate joint 1 counter clockwise by the offset angle
b) Rotate joint 2 clockwise by the offset angle.
The direction for viewing is, in both case, from joint 1 to joint 2.

To determine the turning direction of the offset angle, you always have to take the graphic illustration.
Only in this way is it possible to find the right direction of rotation and to determine whether the offset angle γ_1 and γ_2 have to be summed or have to be subtracted

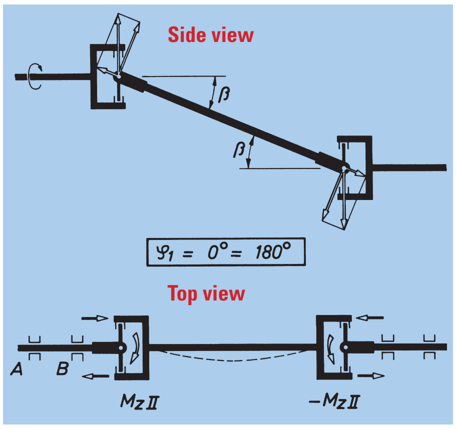
5. Additional moments on the drive line; Bearing loads on the input and output shaft

In Section 2.2 it was shown that the torque is transmitted only in the spider plane and that depending on the fork position, the spider can be perpendicular either to the input axis or the output axis.
What additional forces and moments this causes on the driveline as well as on the bearings of the input and output shaft, is explained briefly in the following chapter.

5.1 With Z-Arrangement

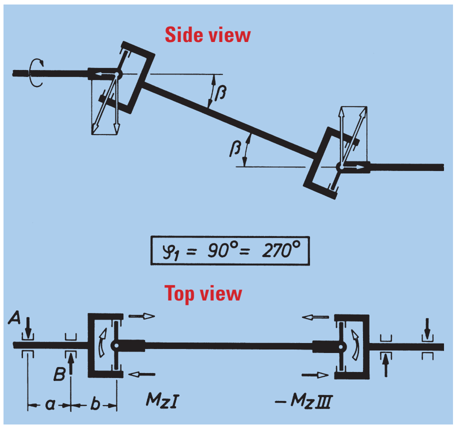
The adjacent illustration shows the location and direction of the additional forces and moments on drivelines having a Z-arrangement, in particular for yoke angles $\varphi_1 = 0°$ and $\varphi_1 = 90°$. This shows clearly, that the driveline center part is stressed by the torque which fluctuates between $M_{dI} \cdot \cos ß$ and $M_{dI} / \cos ß$ in torsion and by the additional periodically alternating, moment M_{ZII} in bending.
(See also Section 6.8).
Likewise, input and output shaft are stressed by M_{ZI} and M_{ZIII} periodically alternating in bending. The resulting bearing loads A and B vary twice per revolution between 0 and maximum value.

Bearing loads on input and output shaft with Z-arrangement



Driveline-center part stressed in bending

$$A = B = 0$$



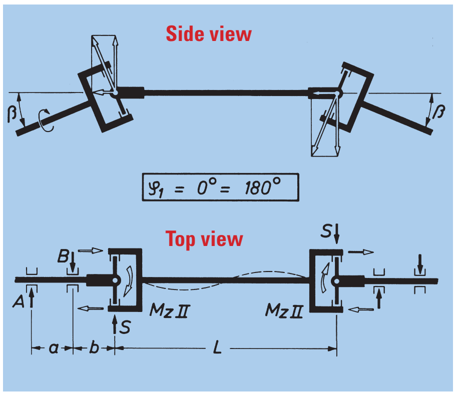
Driveline-center part stressed in bending

$$A_{max} = B_{max} = \frac{Mdl \cdot \tan ß}{a} \quad [N]$$

5.2 With W-Arrangement

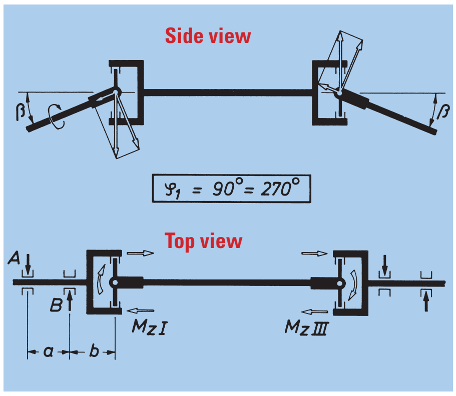
According to the adjacent illustration, with the W-arrangement, an additional force, "S" is introduced, caused by the additional moments M_{ZII} acting in the same direction. The maximum force value occurs at fork position $\varphi_1 = 0°$, and it is transmitted to the input and output shaft by the faces of the spider pins.

Bearing loads on input and output shaft with W-arrangement



Input and output shaft stressed in bending

$$A = \frac{2 \cdot M_{dI} \cdot \sin ß \cdot b}{L \cdot a} \qquad B = \frac{2 \cdot M_{dI} \cdot \sin ß \cdot (a+b)}{L \cdot a}$$



Input and output shaft stressed in bending

$$A = B = \frac{M_{dI} \cdot \tan ß}{a} \quad [N]$$

5.3 Caused by axial displacement forces

If a driveline with an adjustable spline is being changed in length while under torque, in both cases, Z- or W configuration, addition bearing loads are introduced, resulting from the friction caused in the spline. The axial displacement force Pa responsible for these bearing loads is calculated as follows:

$$P_a = 2 \cdot M_{dI} \cdot \mu \left(\frac{1}{dm} + \frac{\sin ß}{Ü} \right) \quad [N]$$

d_m is the spline pitch diameter, Ü the spline overlap. Depending on configuration and lubrication, the coefficient of friction for steel on steel must be assumed to range from 0.11 to 0.15. Plastic coated splines have considerably better sliding characteristics. Here, the friction value is approximately 0.08. Rilsan coated splines are available from size 0.109 up.

6. Fundamental data for sizing of universal drivelines

To size universal drivelines properly, various conditions and factors must be considered. In view of the multitude of possible applications, exact, generally valid rules cannot be provided. The following information is therefore used for the first rough determination of size. In case of doubt, we will gladly compute the required joint sizes for you and, in this context, we like to refer to the technical questionnaires starting on page 189.

6.1 Torques

The max. permitted torques Md_{max} stated for the individual drive-shaft sizes apply normally only for short-term peak loads.
Md_{nom}: Nominal torque for pre-selection on the basis of the operating moment.
Md_{lim}: Limit torque that may be transmitted temporarily from the universal-drive-joint at limited frequency without functional damage.

The respective permissible torque has to be calculated individually depending on the remaining operating data, such as shock loads, angle of deflection, rotation,etc. (See item 6.2 and 6.3)

6.2 Shock loads

Depending on the type of power input or installation, a driveline can be subjected to shock loads considerably above the rated torque. To take those into account, shock service factors must be implemented. Following are some shock-service factors for the most common drives

Prime mover	with flexible coupling	without flexible coupling
Turbine or electric motor	1	1 to 1,5
Gasoline engine, 4 and more cylinders	1,25	1,75
Gasoline engine, 1 to 3 cylinders	1,5	2
Diesel engine, 4 and more cylinders	1,5	2
Diesel engine, 1 to 3 cylinders	2	2,5

Of course, not only the drives, but, in many instances, also the driven equipment is responsible for shock loads. Because of the magnitude of different possibilities, general data valid for every use cannot be supplied.

6.3 Life expectancy – calculation

The decisive factor with regard to life expectancy of universal drivelines is usually the joint bearing. Therefore, in order to determine the individually required joint size, the life expectancy diagram shown later on should be used. This diagram allows to:

a) determine the theoretical life expectancy of a selected driveline as a function of prevailing operating conditions, or
b) to determine the required joint size for a given life expectancy.

In this case, the rated input torque is multiplied by the appropriate service (shock) factor and the M_d such obtained entered in the following diagram. Other factors, such as correction - or deflection angle factor do not have to be considered since they are already incorporated in the diagram.

On machines or vehicles with changing operating conditions, at first, the individual life expectancy values (for each condition) must be determined from the diagram. Then the overall life expectancy L_{hR} can be calculated as follows:

$q_1, q_2 \ldots$ = time share in [%]
$L_{h1}, L_{h2} \ldots$ expressed in 10^3 [Hours]

$$L_{hR} = \frac{100\,000}{\frac{q_1}{L_{h1}} + \frac{q_3}{L_{h2}} + \ldots + \frac{qn}{L_{hn}}} \text{ [Hours]}$$

6.4 Life expectancy-Diagram

In view of the multitude of applications, it is not possible to determine the suitability of a driveline by tests. Therefore, the selection and analysis of the required joint size is done by calculations. These are based on the computation of the dynamic load carrying capacity of full rotation needle - and roller bearings according to ISO recommendation R 281. The life expectancy diagrams shown in the catalogue are based on this recommendation and also on an equation formula especially suited for obtaining nominal life expectancy on universal joints. The thus obtained life expectancy lists the hours of operation that will be reached or exceeded by 90% of a larger number of equivalent universal joint bearings.

There are also methods of obtaining the modified life expectancy. In this case varying survival probabilities, material quality and operating conditions are taken into account. The present technical know how does not allow statements to be made about variations in life expectancy performance resulting from differences in steel quality (grain, hardness, impurities). For this reason, no guidelines have been set in the International Standards.

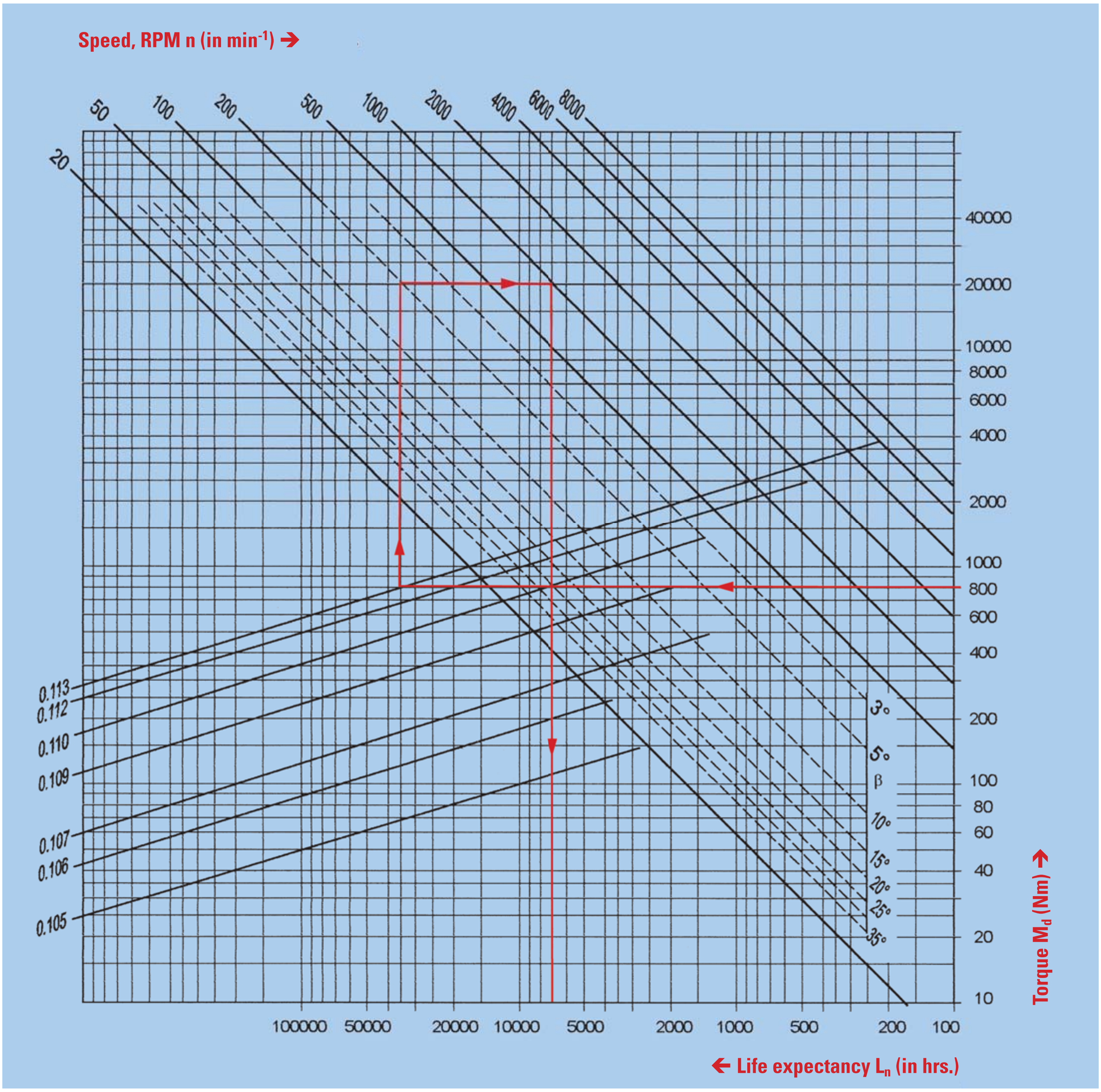
All pertinent operating conditions, such as operating temperature, lubrication intervals, the type of grease used and its viscosity in operation, must also be considered. Since these factors vary from case to case, it is not possible to determine the modified life expectancy and accordingly, a life expectancy diagram valid for universal use.

The two following life expectancy diagrams will allow you to roughly determine the nominal life expectancy.

If the deflection angle is smaller than ß = 3°, ß = 3 should be used. Otherwise, the obtained result will be less accurate.

If it is necessary to determine the life expectancy accurately, kindly consult the ELBE Engineering Department.

6.5 Life expectancy diagram, Needle bearing



Example

Universal driveline 0.113

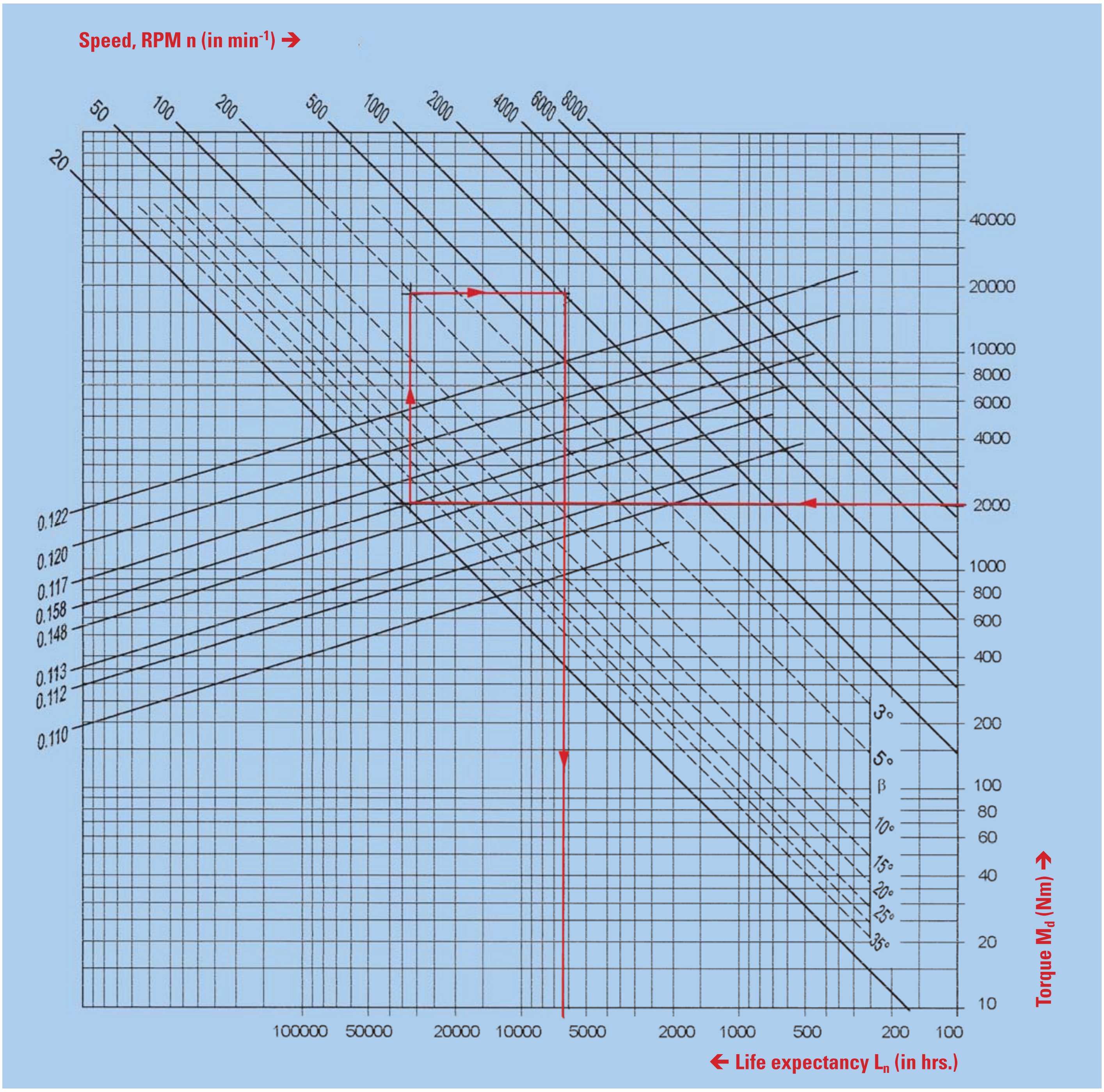
Torque	M_d	=	800 Nm
Deflection angle	ß	=	5°
RPM	n	=	1000 min^{-1}

Life expectancy = 6900 hrs.

Procedure:

Torque ➔ Joint size ➔ Defelction angle ➔ RPM ➔ Life expectancy

6.6 Life expectancy diagram, Roller bearing



Example

Universal driveline 0.158

Torque	M_d	=	2000 Nm
Deflection angle	ß	=	5°
RPM	n	=	1000 min⁻

➔ **Life expectancy = 7000 hrs.**

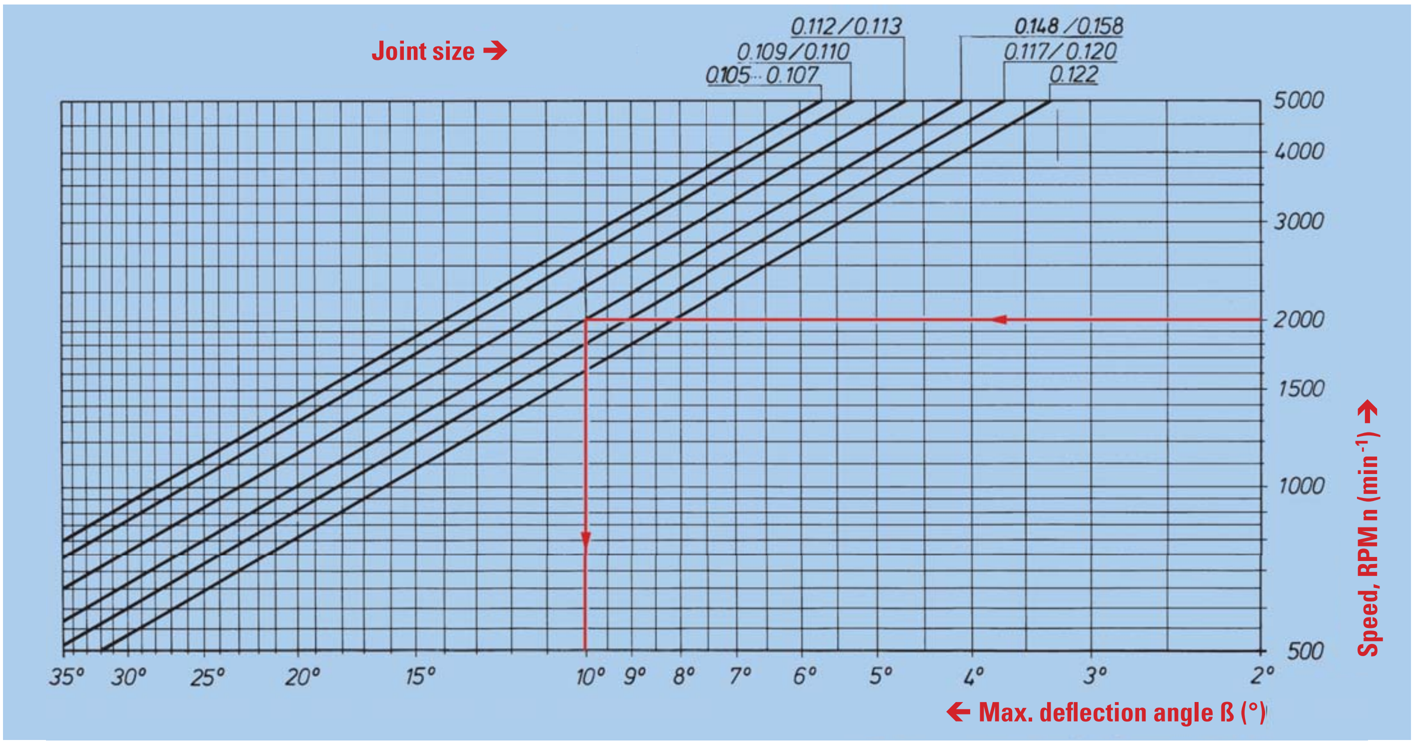
Procedure:

Torque ➔ Joint size ➔ Defelction angle ➔ RPM ➔ Life expectancy

6.7 RPM and deflection angle

As shown in 2.3 by taking certain precautions, a constant output can be obtained on a universal driveline. The center part, however, still retains a non-uniform motion; it is subjected twice per revolution to an acceleration and deceleration. The resulting acceleration torque caused this way is a function of the mass moment of inertia of the driveline's center part as well as of rpm and deflection angle. When regarding smoothness of operation and wear, the product of rpm and deflection angle should not be too high. For use in general mechanical engineering, appropriate guide values can be taken from the diagram below, which is designed for universal drivelines having a standard tubing of up to 1500 mm length.

For vehicle drive trains, these guide values must often be exceeded. Here, at most, up to 1.5 times the diagram value can be permitted.



6.8 Critical speeds

As shown in 5.1, the center part of the angled driveline, when transmitting torque, is stressed periodically in bending by additional moment M_{ZII}. This incites the center part to vibrate. If the frequency of this bending vibration approaches the natural frequency of the driveline, maximum stress in all components, buckling of the shaft and development of noise will result.

To avoid this, long and fast running drivelines must be checked for critical bending vibration speeds. The critical, first order bending vibration speed of a driveline employing tubing can be roughly calculated as follows:

$$n_{kr} \approx 1{,}21 \cdot 10^8 \frac{\sqrt{D^2 + d^2}}{L^2} \quad [\mathrm{min}^{-1}]$$

D = Tubing-outside diameter [mm]
d = Tubing-inside diameter [mm]
L = Center part length in [mm]

Drivelines are used in the subcritical zone only. For reasons of safety, it must be ensured that the maximum operating speed is far enough away from its system's resonance (critical) speed. Therefore, the following applies:

Max. Operating Speed $n_{max} \approx 0{,}65 \cdot n_{cr}$ [RPM]

6.9 Larger tubing diameters

The critical bending vibration speed of a driveline is, as can be seen from the critical rpm formula, a function of tubing diameters and length of center part. By going to larger tubing diameters, the critical speed of a driveline can be increased. However, the diameter increase must remain within defined limits since a certain relationship between tubing dimensions and joint size must be adhered to.

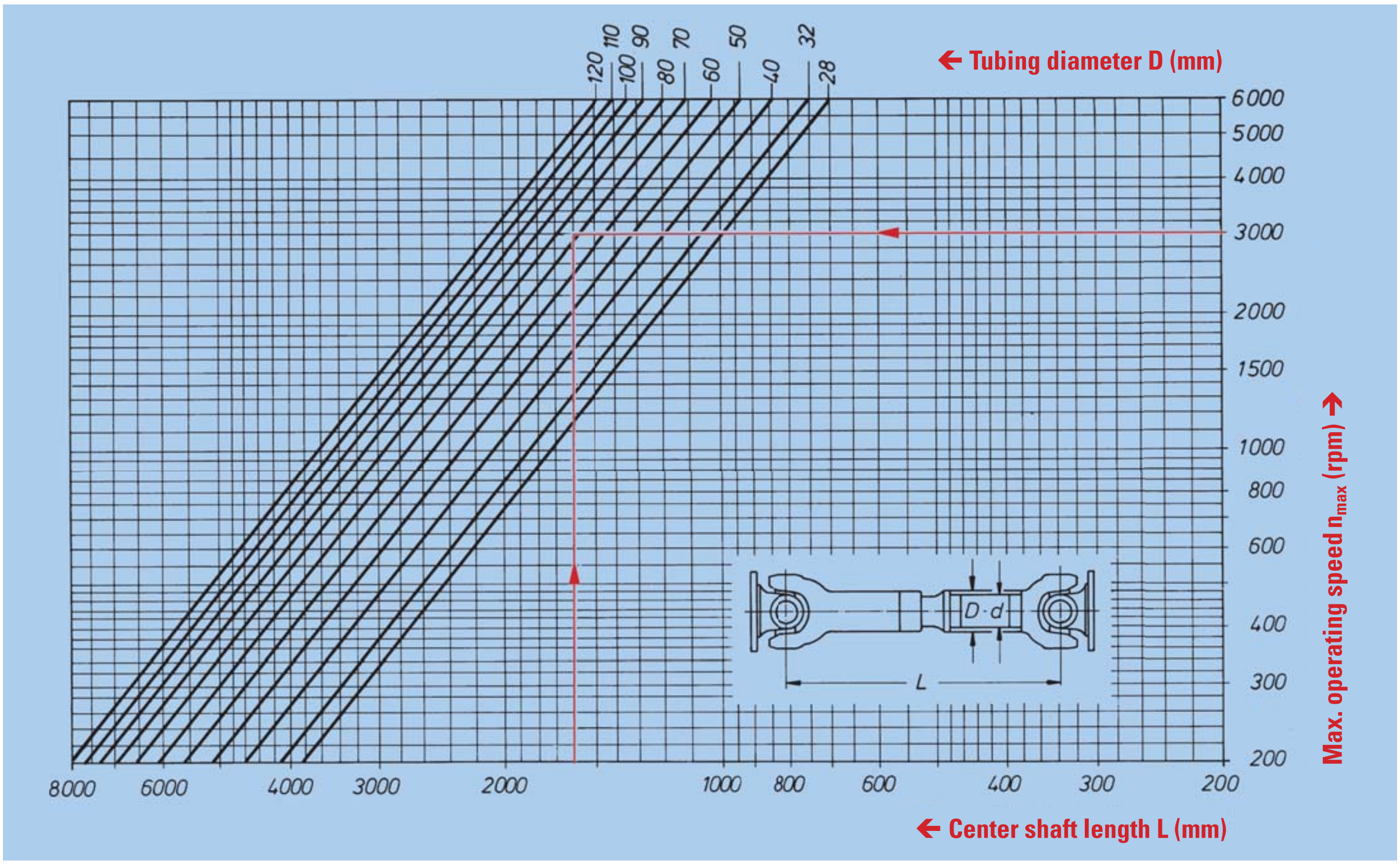
The dimension sheets of the different driveline models list the possible tubing dimensions for each size. In all the cases where a single driveline is insufficient, multiple arrangements with intermediate bearings must be used.

It must be noted that larger tubing diameters are feasible only above a certain shaft length. The following minimum lengths can be used as an angle line.

Flange diameter	[mm]	Up to 65	75 to 100	120 to 180
Min. length S	[mm]	650	950	1250

6.10 Tubing diagram

For determining the required tubing diameter when maximum operating speed n_{max} and center part length L are given.



Example:

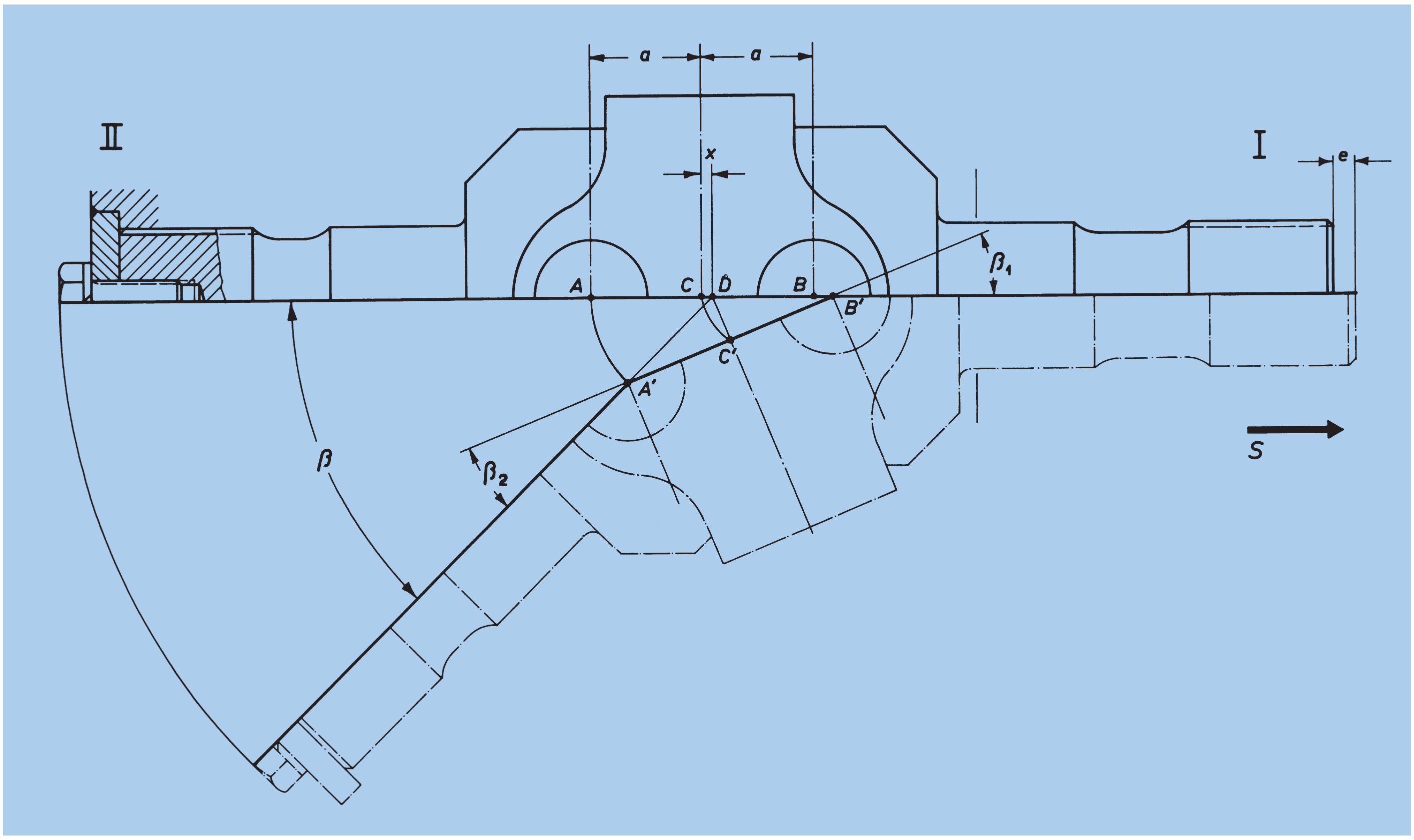
Center shaft length L = 1600 mm
Max. operating speed n_{max} = 3000 RPM
} Obtained: Tubing diameter ≧ 70 mm

7. Application principles for double joint shafts in steering axles

The double joint shafts of series 0.400.5 and 0.500.3 are intended for use in powered steering axles only.

7.1 Kinematic conditions

As shown in the sketch below, when steering is activated, the axle system is rotated around pin center **D**. The double joint deflects at its two joint pivot points **A** and **B**. Since shaft II is fixed axially, shaft I must move in the direction **S**. This causes unequal joint deflection angles **ß$_1$** and **ß$_2$**, and therefore, also a non-uniform (fluctuating) output motion. The fluctuation can be kept very small provided joint center **C** is offset toward the fixed side by the compensation value X. This way, at a certain deflection angle (= synchronous motion angle **ß$_x$**) completely uniform motion is obtained, i.e., the two joint deflection angles **ß$_1$** and **ß$_2$** are equal.
ßx = 30° bis 35° would be an appropriate synchronous motion angle to select



A B	=	Joint pivot point
C	=	center of the double joint
D	=	rotation pin center
a	=	distance of a joint point from the center ot the double joint
e	=	axial movement of floating shaft
X	=	center offset on installation
ß$_x$	=	uniform motion angle (synchronous)
ß	=	total deflection angle
ß$_1$ ß$_2$	=	deflection angle of each individual joint

7.2 Center offset value x and max. slide movement e

The center offset **X** required for smooth output can be derived from distance a and synchronous motion angle ß:

$$X = \frac{a}{\cos \frac{ß_x}{2}} - a$$

Calculated center offset value **X** for individual joint sizes:
Series 0.400, synchronous motion angle $ß_x$ = 35°

Joint size	0.408	0.409	0.411	0.412
Deflection angle ß °	50	50	50	50
x [mm]	1,5	1,7	2,0	2,2

Series 0.500, synchronous motion angle $ß_x$ = 32°

Joint size	0.509	0.510	0.511	0.512	0.513	0.515	0.516	0.518
Deflection angle ß °	42 | 47	| 50	42 | 47	42 | 47	42 | 47	42 | 47	42 | 47	42 | 47
x [mm]	1,3 | 1,3	| 1,6	1,5 | 1,6	1,6 | 1,7	1,7 | 1,8	1,9 | 2,0	2,1 | 2,2	2,2 | 2,3

Sliding motion **e** at deflection angle ß, and also as a function of distance a and uniform motion angle $ß_x$, can be calculated as follows:

$$e = 2a \left(\frac{\sin^2 \frac{ß}{2} + \sqrt{\cos^2 \frac{ßx}{2} - \sin^2 \frac{ß}{2} \cdot \cos^2 \frac{ß}{2}}}{\cos \frac{ßx}{2}} - 1 \right)$$

Max. slide motion **e** for the individual joint sizes:
Series 0.400, synchronous motion angle $ß_x$ = 35°

Joint size	0.408	0.409	0.411	0.412
Deflection angle ß °	50	50	50	50
e [mm]	6,5	7,2	8,3	9,2

Series 0.500, uniform synchronous motion angle $ß_x$ = 32°

Joint size	0.509	0.510	0.511	0.512	0.513	0.515	0.516	0.518
Deflection angle ß °	42 | 47	| 50	42 | 47	42 | 47	42 | 47	42 | 47	42 | 47	42 | 47
e [mm]	4,5 | 6,0	| 7,9	5,2 | 6,9	5,8 | 7,8	6,1 | 8,1	6,7 | 9,0	7,3 | 9,7	7,8 | 10,5

7.3 Sizing of double joint shafts

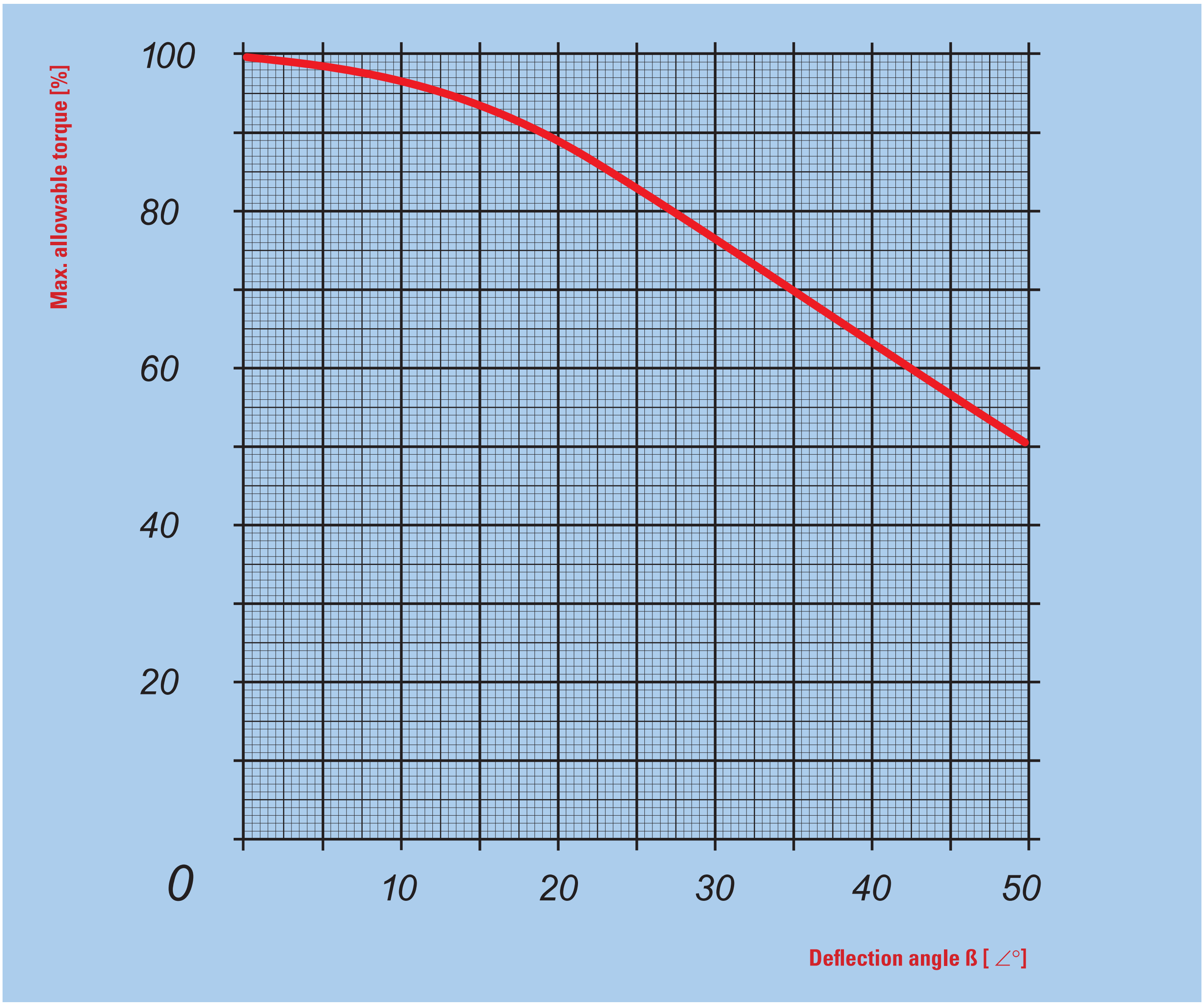
Max. possible torque should be used for determining the required joint size. This could be the input torque, calculated from prime mover output, gear ratio and power distribution, or also the tire slippage torque, derived from allowable axle loading, static tire radius and coefficient of friction. The lower of the two values represents the maximum operating torque which should be used for determining the proper joint size. The double joint shaft selected this way will have adequate life expectancy, since the time percentage of maximum loading is usually low.

7.4 Loads on the shaft bearing

Double joint shafts, when not centered, must have a bearing support at both shaft halves right next to the joint with one shaft half fixed axially and the other floating axially. When torque is being transmitted, additional forces occur which must be taken into account when sizing the bearings.

7.5 Torque capacity of double joints as a function of deflection angle

Under torque, different force conditions exist at the joint spider pins and center piece with the double joint in an angled position than in a straight position. The reason for this is that the torque to be transmitted is not distributed evenly over the joint spider pins any longer. Also, as mentioned in Chapter 5, an additional moment occurs. This additional moment must be combined with the torque to be transmitted. This resulting moment leads to higher compression loads and to a larger bending stress within the joint spider pins. The diagram below allows to take these factors into account. It shows the percentage the maximum allowable torque must be reduced in relation to the deflection angle.

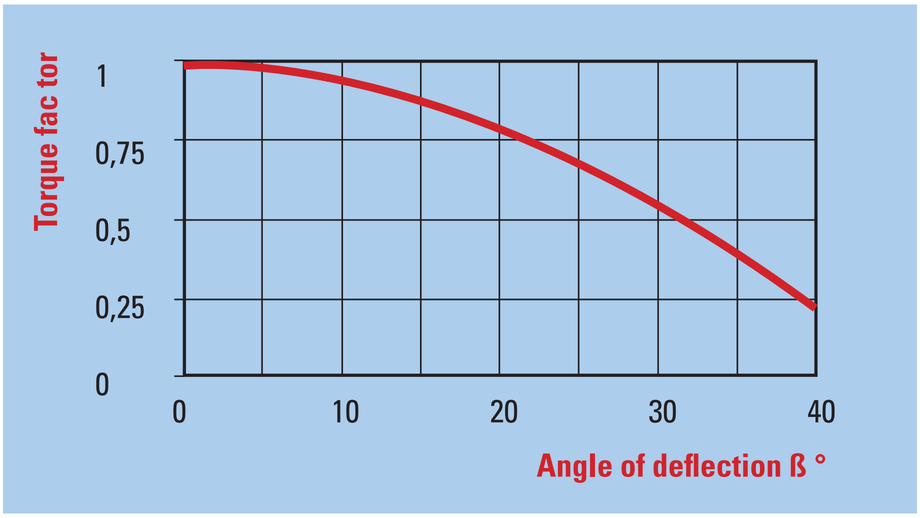


8. Hints for the application of pin and block cardan joints, ball and socket cardan joints

Torque calculation for needle bearing equipped precision cardan shafts, pin and block cardan joints, ball and socket cardan joints, single

The values Md_{max} listed in the diagram represent limit values that may not be exceeded. They are admissible to the full extent only at small rotation speed and minor angle of deflection respectively during intermittent operation.

The transmissible torque varies depending on the size of the angle of deflection.



Needle bearing equipped precision cardan joints

Permitted max. operation moments of the needle bearing equipped precision cardan joints (Torque in Nm)..

Joint type	250	500	Speed (r.p.m.) 1000	2000	3000	4000	5000
0.616	11	10	8	6	5,5	5,1	4,8
0.620	28	25	19	15	14	12,5	12
0.625	35	30	25	20	18,5	17	16
0.632	70	60	50	40	37	34	32
0.640	150	130	100	80	74	68	64
0.650	220	190	150	120	110	100	95
0.663	450	400	310	250	220	200	190

Pin and Block cardan joint, Ball and socket cardan joint, single

The empirical formula on the right can be used for the rolled calculation of the required joint size.

At	Md_{max}	Speed x bending angle ≤ 500
At 0,5 x	Md_{max}	Speed x bending angle ≤ 5000

Recommendations for maintenance

An adequate lubrication shall be ensured for universal and ball-and-socket joints in permanent operation. Where drip oiling is not feasible, the joints have to be once daily lubricated (for lubricating points see arrow). Joints may also be enveloped in bellows; such bellows for these tow joint types may be ordered from us.

For utilization of cardan shafts under extreme climatical conditions (high and low temperatures) consult us first.